

**ACTIVE VIBRATION CONTROL OF PIEZO STACK
ACTUATOR WITH CONSIDERATION TO
HYSTERESIS AND SATURATION EFFECTS**

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UNIVERSITI SAINS MALAYSIA

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ACTUATOR WITH CONSIDERATION TO HYSTERESIS AND
SATURATION EFFECTS**

by

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Thesis submitted in fulfillment of the requirements

for the degree of

Doctor of Philosophy

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DECLARATION

I hereby declare that the work reported in this thesis is the result of my own investigation and that no part of the thesis has been plagiarized from external sources. Materials taken from other sources are duly acknowledged by giving explicit references.

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LIST OF ABBREVIATIONS

Symbols	Descriptions
AFC	Active Force Control
AVC	Active vibration control
BC	Back-calculation
CTS	Carpal Tunnel Syndrome
DVA	Dynamic vibration absorber
EAV	Exposure Action Value
ELV	Exposure Limit Value
EM	Electromagnetic
EU	European Union
FFT	Fast Fourier Transform
FRF	Frequency response function
HAV	Hand-arm vibration
HVA	Hybrid vibration absorber
iMC	Integrated measurement and control
ISO	International Organization for Standardization
ISP	Integral state prediction
LQG	Linear Quadratic Gaussian
NI	National Instruments
NSC	Nested Switching Control
PID	Proportional-integral-derivative
PSC	Parallel stiffness combination
PTFE	Poly Tetra Fluoro Ethylene
PVDF	Poly Vinylidene Fluoride

PWM	Pulse width modulated
PZT	Lead Zirconate Titanate
RMS	Root mean square
SDOF	Single-degree-of-freedom
SDM	Structural dynamic modification
SMC	Sliding mode control
TMC	Technical Manufacturing Corporation
TMD	Tuned mass damper
TVA	Tuned vibration absorber
VWF	Vibration White Finger

LIST OF SYMBOLS

Symbols	Descriptions	Unit
A	System matrix	-
A_m	SDM system matrix	-
$A(x)$	Beam cross section area	m^2
a_s	Saturation lower limit value	V
a_h	First order piezo hysteresis constant	-
a_{hv}	Frequency-weighted acceleration	m/s^2
a_l	First order piezo linear constant	-
α_P	Preisach hysteresis switching up value	V
$\alpha_{(V,\omega)}$	Piezo proportional coefficient	-
$\Delta\alpha_{(V,\omega)}$	Piezo derivative part (Hysteresis effect)	-
$\Delta\alpha^{-1}_{(V,\omega)}$	Feed-forward inverse piezo hysteresis model	-
B	Input matrix	-
B_m	SDM input matrix	-
b	Beam width	m
b_h	First order piezo hysteresis constant	-
b_l	Piezo linear constant	-
b_s	Saturation upper limit value	V
$b_{(F,\omega)}$	Inverse piezo proportional coefficient	-
$\Delta b_{(F,\omega)}$	Inverse piezo derivative part (Hysteresis effect)	-
β	Load cell force sensitivity	N/V
β_P	Preisach hysteresis switching down value	V
c	Dynamic damping of the handle	kg/s
c_l	Piezo hysteresis constant	-

c_m	Modified dynamic damping of the handle	kg/s
Δc	Changes of handle dynamic damping	kg/s
C	Damping matrix	kg/s
C_m	Modified modal damping matrix	kg/s
ΔC	Changes of damping matrix	kg/s
C_a	Piezo stack actuator capacitance	nF
d	System disturbance	-
d_m	SDM system disturbance	-
D_3	Piezo electrical displacement	C/m ²
d_{33}	Piezo charge constant	m/V
E	Beam Young's modulus of elasticity	N/m ²
ΔE	Changes of beam Young's modulus of elasticity	N/m ²
e	PID controller error signal	m
e_I	Integral error signal	m
E_3	Piezo electrical field	V/m
ϵ_{33}	Piezo dielectric constant	F/m
f_{\max}	Maximum frequency range	Hz
f_{\min}	Minimum frequency range	Hz
f^P	Preisach hysteresis model output	N
f^{PI}	Prandtl-Ishlinski hysteresis model output	N
$F(j\omega)$	Excitation force	N
$F_a / F_{a \text{ dyn}}$	Piezo stack actuator force	N
$F_{ai \text{ hys}}$	Piezo stack actuator force (Hysteresis)	N
$F_{ai \text{ lin}}$	Piezo stack actuator force (Linerized)	N
F_{am}	Piezo stack actuator force (SDM effect)	N

$F_{a\ max}$	Piezo stack actuator maximum force	N
$F_{b\ max}$	Piezo stack actuator maximum blocking force	N
F_d	Disturbance force	N
F_L	Piezo stack actuator preload	N
$F_{L\ max}$	Maximum piezo stack actuator preload	N
$F_L(x,t)$	Beam external preload	N
F_r	Friction induced force	N
F^*	Estimated disturbance force	N
f	Force vector	N
g	Gravity acceleration	m/s ²
G	Output matrix	-
G_m	SDM output matrix	-
h	Beam thickness	m
Δh	Changes of beam thickness	m
I_{BC}	Integral gain of back-calculation anti-windup	-
I_{Clamp}	Integral gain of clamping anti-windup	-
I_{Linear}	Integral gain of PID controller with linearized piezo	-
I_{Track}	Integral gain of tracking mode anti-windup	-
$I(x)$	Beam area moment of inertia	kg m ²
i	Prandtl-Ishlinski hysteresis play operator index	-
i_a	Piezo amplifier current	A
$i_{a\ max}$	Maximum piezo amplifier current	mA
k	Dynamic stiffness of the handle	N/m
Δk	Changes of handle dynamic stiffness	N/m
k_a	Piezo stack actuator stiffness	N/m

k_{AFC}	AFC percentage constant	-
K_B	Back-calculation gain	-
K_D	Derivative gain	-
$K_{D \text{ hys}}$	Derivative gain (Hysteresis)	-
$K_{D \text{ lin}}$	Derivative gain (Linearized)	-
k_{exist}	Existing lower handle stiffness	N/m
K_I	Integral gain	-
$K_{I \text{ hys}}$	Integral gain (Hysteresis)	-
$K_{I \text{ lin}}$	Integral gain (Linearized)	-
k_m	Modified dynamic stiffness of the handle	N/m
k_{max}	Beam stiffness upper limit	N/m
k_{min}	Beam stiffness lower limit	N/m
K_P	Proportional gain	-
$K_{P \text{ hys}}$	Proportional gain (Hysteresis)	-
$K_{P \text{ lin}}$	Proportional gain (Linearized)	-
$K_{P \text{ step}}$	Proportional step up gain	-
$k_{s \text{ total}}$	Total spring stiffness	N/m
k_{s1}	Spring stiffness 1	N/m
k_{s2}	Spring stiffness 2	N/m
K_T	Tracking mode gain	-
Δk_{m1}	Changes of lower handle stiffness 1 (Aluminium)	N/m
Δk_{m2}	Changes of lower handle stiffness 2 (Mild steel)	N/m
\mathbf{K}	Stiffness matrix	N/m
$\Delta \mathbf{K}$	Changes of stiffness matrix	N/m
\mathbf{K}_m	Modified modal stiffness matrix	N/m

Δk_{n1}	Core stiffness 1 (Centre)	N/m
Δk_{n2}	Core stiffness 2 (Centre-end)	N/m
Δk_{n3}	Core stiffness 3 (End-end)	N/m
Δk_{n4}	Parallel stiffness 1 ($\Delta k_{n1} + \Delta k_{n2}$)	N/m
Δk_{n5}	Parallel stiffness 2 ($\Delta k_{n1} + \Delta k_{n3}$)	N/m
Δk_{n6}	Parallel stiffness 3 ($\Delta k_{n2} + \Delta k_{n3}$)	N/m
Δk_{n7}	Parallel stiffness 4 ($\Delta k_{n1} + \Delta k_{n2} + \Delta k_{n3}$)	N/m
L	Piezo stack actuator length	mm
l	Beam length	m
M	Additional load	kg
$M(j\omega)$	Mobility FRF	m/s N
$M_c(j\omega)$	Mobility damping	m/s N
$M_{cm}(j\omega)$	Modified mobility damping	m/s N
$M_k(j\omega)$	Mobility stiffness	m/s N
$M_{km}(j\omega)$	Modified mobility stiffness	m/s N
$M_m(j\omega)$	Mobility mass	m/s N
$M_{mm}(j\omega)$	Modified mobility mass	m/s N
$M_{SDM}(j\omega)$	Modified mobility FRF	m/s N
m	Dynamic mass of the handle	kg
Δm	Changes of handle dynamic mass	kg
m_0	Unbalance mass (Collet)	kg
m_a	Mass of the piezo stack actuator	kg
m_{eff}'	Effective mass	kg
m_m	Modified dynamic mass of the handle	kg
M^*	Estimated mass	kg